Collective Effects in dE/dx:

Attaining Extreme Parameters in Warm Dense Matter

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With critical help from Dr. Ilya Dodin and Dr. Vladimir Malkin

Can Warm Dense Matter be utilized uniquely to attain fundamentally new effects? Can Warm Dense Matter be utilized to reach extremes in parameters, such as particle acceleration energies, acceleration gradients, differential stopping powers, ultra-high magnetic fields, or ultra-high x-ray intensities?

Much of the use of ideal plasma for extreme effects, including for accelerating charged particles or compressing light, is mediated through the use of the plasma wave. In achieving similar effects in dense plasma, how much of the plasma wave need survive?

Overarching Question

Challenging theory, simulations, and experiments …

But, just suppose we understood Warm Dense Matter a lot better …

What could possibly be the solution space of the absolutely compelling applications?

Outline (some possible solution-space features)

Attaining New Effects or Extreme Parameters in Warm Dense Plasma

1. Acceleration of charged particles through plasma wakes
   a. What are maximum gradients and energies?
   b. How long need a plasma wave persist?
2. Deceleration of charged particles through plasma wakes
   a. Fast ignition with differential core stopping
3. Current Drive in Dense Plasma
   a. What is highest magnetic field that can be generated in the laboratory?
   b. Diagnostic of Basic Physics
4. X-ray pulse compression and focusing; fs to as
   a. What are the largest x-ray intensities that can be obtained and focused -- vacuum breakdown?
   b. How long need a plasma wave persist?

All Effects Mediated by Plasma Wave!

Plasma Oscillations in Ideal Plasma

$$\nabla \bullet \vec{E} = 4\pi e(n_0 - n_e) = -4\pi e\tilde{n}$$

Poisson's equation

$$\frac{\partial}{\partial t} n_e + \nabla \cdot n_e v = 0$$

Particle conservation

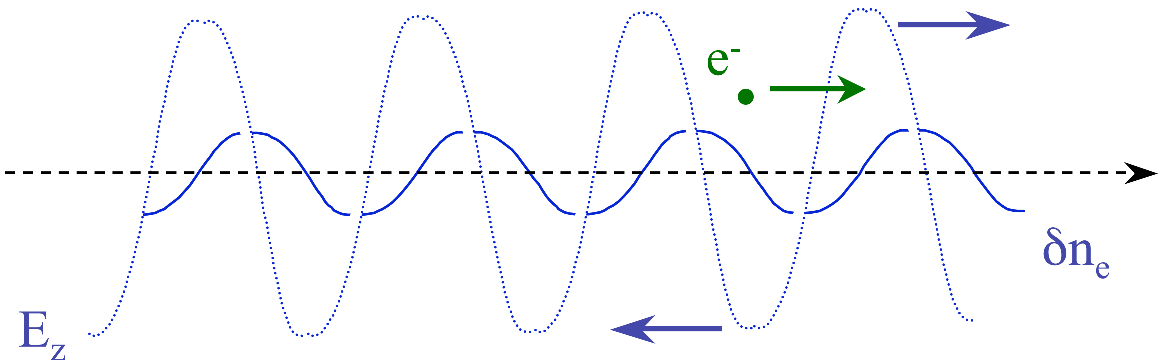
$$\frac{\partial}{\partial t} n_e m v + \nabla \cdot n_e m v v = eE$$

Momentum conservation

$$\Rightarrow \quad \boxed{\frac{\partial^2}{\partial t^2}\tilde{n} + \omega_p^2 \tilde{n} = 0}$$

$$\boxed{\tilde{n} = A(\vec{r})\cos\omega_p t + B(\vec{r})\sin\omega_p t}$$

Electron acceleration in a plasma wave

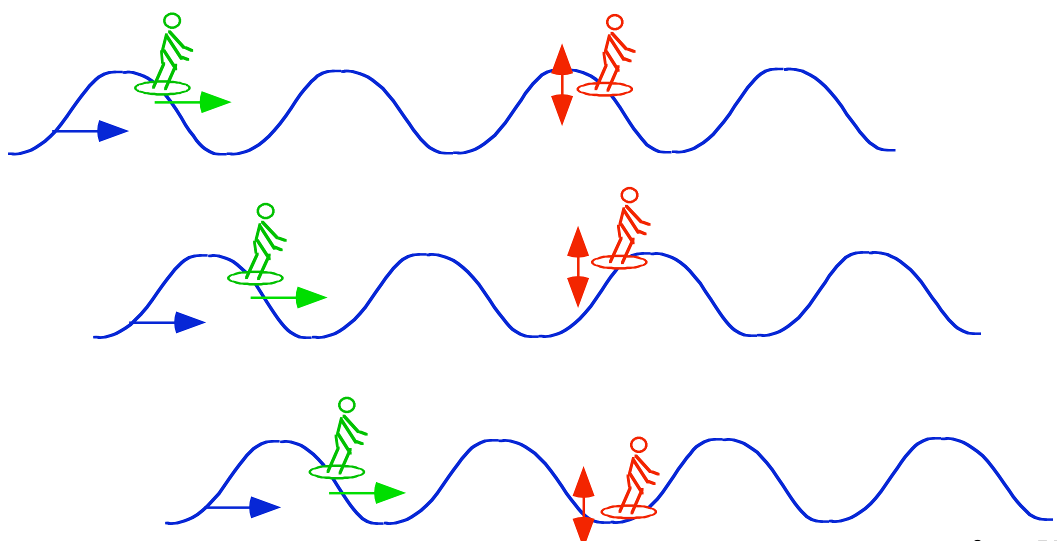


phase velocity -- arbitrary

Accelerate to TeV

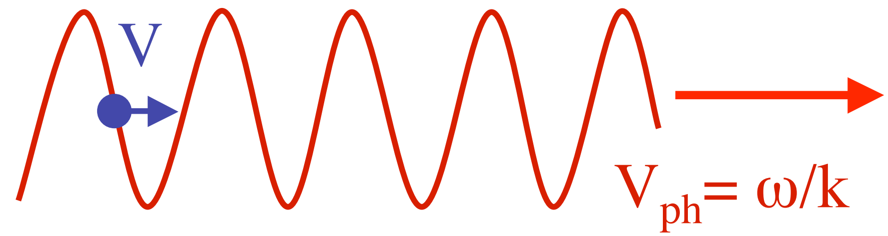
Tajima and Dawson (1979)

Analogy:



from V.Malka

Accelerating Gradient in Plasma



Conventional Accelerator

Gradients ~ 20 MeV/m at 3GHz

1 TeV Collider requires 50 km

Peak gradients limited by breakdown

$$\nabla \bullet \vec{E} = -4\pi e\tilde{n}$$

Plasma Accelerator

High fields, No breakdown

(Tajima and Dawson, 1979)

$$\tilde{n}_{MAX} \approx n_0$$

$$k = \frac{\omega_p}{c}$$

$$eE_{MAX} \approx \sqrt{n_0} GeV/cm$$

Example

$n_0 = 10^{18} cm^{-3}$

$eE = 100$ GeV/m

Note: For v << c , $\frac{v_{osc}}{c} \approx \frac{\tilde{n}}{n_0}$

Particles accelerated to relativistic energies, even as plasma motion is not

Accelerating Gradient in VERY DENSE Plasma

- Accelerating gradient increases with the plasma density.
- In solid state, perhaps plasma waves can accelerate charged particles to TeV energies in only a few centimeters.

Ideal Plasma

$n_0 = 10^{18} \mathrm{cm}^{-3}$

$eE = 100$ GeV/m

1 TeV accelerator requires 10 m

$$\nabla \bullet \vec{E} = -4\pi e \tilde{n}$$

$$\tilde{n}_{MAX} \approx n_0$$

$$k = \frac{\omega_p}{c}$$

$$eE_{MAX} \approx \sqrt{n_{24}}\, TeV/cm$$

Limitations on the Maximum Accelerating Gradient

- Accelerating force scales like $eE \propto \sqrt{n}$
- Friction $F = -\nu p$ scales like $F \propto n$
- The limiting density is given by $eE(n) + F(n) = 0$

$$\boxed{n_{\max} \sim \frac{4\pi e^2 m_e c^2}{p^2 \overline{\nu}^2}}$$

$$\boxed{E_{\max} \sim \sqrt{4\pi m_e c^2 n_{\max}}}$$

$$p = \gamma m v, \quad \overline{\nu} = \nu / n$$

Slowing-down (Frictional) Forces

Coulomb Collisions

$$\frac{\Delta \mathcal{E}}{\Delta z} = -\frac{1}{v\tau}\frac{\delta p^2}{2M}$$

$$\delta p^2/\tau \sim \nu_\perp p^2$$

$$\frac{d\mathcal{E}}{dz} \sim -\frac{\nu_\perp p^2}{Mv}$$

$$\frac{d\mathcal{E}}{dz} = -\frac{4\pi N Z^2 e^4 \Lambda}{Mc^2 p} = -\nu\mathcal{E}$$

Energy decays primarily by scattering on electrons due to small *M.*

Bremsstrahlung

$$\nu = cX_0^{-1}$$

$$\boxed{X_0^{-1} \approx 4\alpha N_a Z_a^2 r_e^2 \left(\frac{m_e}{m}\right)^2 \ln\left(184\, Z_a^{-1/3}\right)}$$

$$\alpha = e^2/\hbar c \approx 1/137$$

$$r_e = e^2/m_e c^2 \approx 2.82\times 10^{-13}\,\mathrm{cm}$$

Bremsstrahlung is most efficient on ions because of their high *Z*.

Rossi (1941), Tsai (1974), Roos et al (1982), Klein (1999), Uggerhøj (2005)

Maximum Density and Accelerating Gradient

$$n_{\max} \sim \frac{10^{40}}{\gamma^2 Z_a^4}\left(\frac{m}{m_e}\right)^2 \ \mathrm{cm}^{-3}$$

$$eE_{\max} \sim \frac{10^8}{\gamma Z_a^2}\left(\frac{m}{m_e}\right)\frac{\mathrm{TeV}}{\mathrm{cm}}$$

$$\mathcal{E}_{\max} \sim \frac{1}{2Z_a^2\sqrt{n_{22}}}\left(\frac{m}{m_e}\right)^2 \ \mathrm{PeV}$$

e.g.: $n \sim 10^{22}\ \mathrm{cm}^{-3}$,

$Z_a \sim 40$,

$eE \sim 100\ \mathrm{GeV/cm}$,

$\mathcal{E}_e \sim 300\ \mathrm{GeV}$,

$\mathcal{E}_p \sim 10^6\ \mathrm{TeV}$

Pitch-angle Scattering and Channeling

Bremsstrahlung leads to pitch-angle scattering:

$$\boxed{\frac{d\left\langle \delta\theta^2 \right\rangle}{dz} \approx \frac{4\pi}{\alpha\gamma^2 X_0}}$$

Electrons with $\gamma^2 \sim 4\pi/\alpha$, or $\mathcal{E} \sim 20$ MeV, get $\delta\theta \sim 1$ within few centimeters.

Rossi (1941), Highland (1975), Hansen (2003)

Pitch-angle scattering results in the particle transverse escape from the tightly focused accelerating field; hence the acceleration stops.

Therefore, particles must be channeled during the acceleration.

Channeling in Crystals

$$m\gamma\ddot{x} + \underbrace{\dot{\gamma}\dot{x}}_{\substack{\text{adiabatic}\\ \text{mass increase}}} + \underbrace{Kx}_{\text{focusing}} = \underbrace{\delta\dot{p}(t)}_{\substack{\text{pitch-angle}\\ \text{scattering}}} + \underbrace{\beta\dot{x}}_{\substack{\text{radiation}\\ \text{losses}}}$$

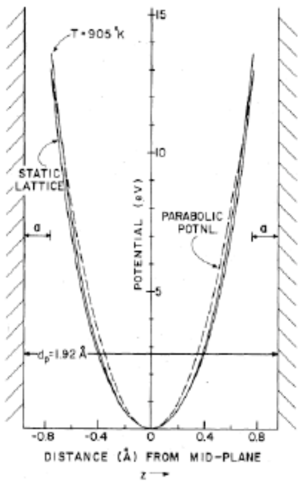


FIG. 9. Continuum potential energy for protons channeled in the (110) planes of Si. The calculations are based on Molière's ion–atom potential One of the solid curves is for the static case, with U_{PS} given by Eq. (2.31); the other is for a temperature of 905 °K, with U_{PV} derived from V_{PV} given in Eq. (2.75). For comparison, the dashed curve is a simple parabolic potential set equal to the value of U_{PS} at $z = (\frac{1}{2}d_p - a)$.

from Gemmel (1974)

$$\boxed{\Delta\left\langle x^2 \right\rangle \propto \frac{1}{eE}\left(1 - \sqrt{\frac{\gamma_0}{\gamma(t)}}\right) \longrightarrow \text{const}}$$

- Channeling persists at conceivable $E > 10\,\frac{\text{GV}}{\text{cm}}$
- Channeling radiation:

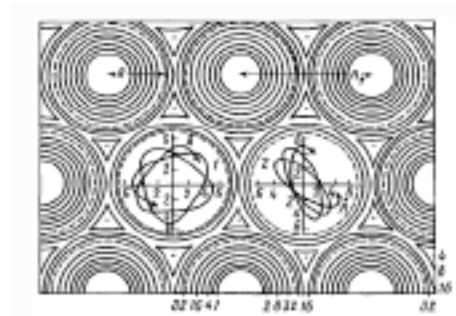
$$\boxed{P_\beta = -\frac{2}{3}\frac{e^2K}{m^2c^3}\gamma^2\left\langle x^2 \right\rangle}$$

Chen and Noble, 87

Other Channeling Concepts

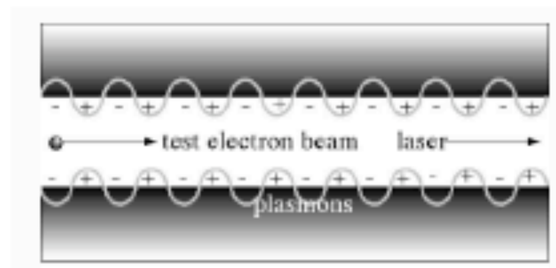
- Channeling in nanotubes (C_{60}, porous Si, artificial channels)

  Gevorgyan *et al* (1997), Newberger *et al* (1989), Tajima (2003)



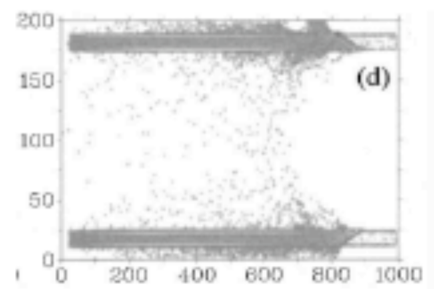
- Interaction with surface: still high density, yet accessible with optical lasers

  Saito and Ogata (2003)



- Rapid plasma expansion inside an optical capillary (10 GeV/m, tens of MeV)

  Bulanov *et al* (1994), Rau and Caims (2000)



Maximum Energy Gain with Channeling in a Crystal

$$\frac{d\mathcal{E}}{dt} = \underbrace{ecE}_{\text{acceleration}} - \underbrace{P_b}_{\text{bremmsstrahlung}} - \underbrace{P_i}_{\text{ionization}} - \underbrace{P_\beta}_{\substack{\text{channeling}\\ \text{radiation}}}$$

- Equation for the maximum energy:

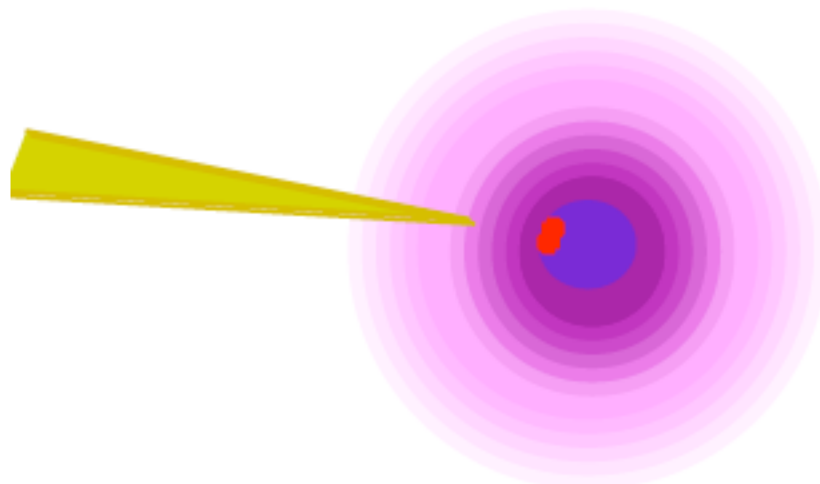
$$\boxed{\left(P_b + P_i + P_\beta\right)_{\max} = ecE}$$

$$P_i \ll P_\beta \sim P_b \approx ecE \sim 100c \text{ GeV/cm}$$

$$\mathcal{E}_{e^+} \sim 300 \text{ GeV}, \quad \mathcal{E}_p \sim 10^6 \text{ TeV}$$

“Deceleration” in Warm Dense Matter

Using collective effects



Fast ignitor
~100kJ, 10 ps, to ~10 keV
~20 µm spot size

Optical laser pulses propagate just at densities1000 – 10000 times smaller than the core density.

Depositing energy with MeV electrons

Conventional paradigm: electron beams heat the core primarily by Coulomb collisions.

However just electrons of energy not much higher than 1 MeV can be stopped in the core.

Can Langmuir waves help to decelerate even ultra-relativistic electrons in the core?

Advantages:

More intense lasers can be used.
More energy can be deposited in smaller volume and shorter times.
More energy can be transported at less current, reducing instabilities.

Collisional threshold for the turbulence excitation

Collisions between the core electrons and ions cause strong damping of Langmuir waves. To overcome it, the collisionless instability, the growth rate must exceed the damping rate:

$$\Gamma > \nu$$

$$n_e = 10^{26}\ \mathrm{cm}^{-3} \quad \longrightarrow \quad \boxed{\nu \approx 5\times 10^{14}\ \mathrm{sec}^{-1}}$$

$$T_e \sim 5\ \mathrm{KeV}$$

Ideal Plasma: Collisionless Instability Growth Rate

In the kinetic instability regime, the maximized over Langmuir wavevectors collisionless growth rate is

$$\boxed{\Gamma \sim \frac{2\omega_e n_b}{\gamma_b \Delta\vartheta^2 n_e},}$$

where $\omega_e = 5.64\times10^4 \sqrt{n_e \times \mathrm{cm}^3}\ \sec^{-1}$ – is electron plasma frequency; n_b, γ_b and $\Delta\vartheta$ are the beam concentration, relativistic factor and angular spread, respectively.

Numerical example :

$n_e = 10^{26}\ \mathrm{cm}^{-3}$, $n_b = 5\times10^{22}\ \mathrm{cm}^{-3}$, $\Delta\vartheta = 0.1$ and $\gamma_b = 50$,

gives

$$\boxed{\Gamma \sim 10^{15} \sec^{-1}}$$

Beam transport near the core

$$\Gamma \propto \omega_e \frac{n_b}{n_e} \propto n_e^{-1/2}, \quad \nu \propto n_e T_e^{-3/2} \text{ (up to a logarithm)}$$

$$\Gamma / \nu \propto (T_e / n_e)^{3/2}.$$

If T_e/n_e decreases outside, the core instability can coexist with the stable transport near the core.

If T_e/n_e increases outside, another effect would be needed to stabilize the transport near the unstable core, so that the power can reach the core.

Inhomogeneous Plasma

Ray optics

$$\psi = \int exp[\imath S(x,t)] \qquad S \gg 1$$

$$\frac{\partial S}{\partial x} = k \qquad \frac{\partial S}{\partial t} = -\omega(k,x,t)$$ Hamilton–Jacobi equation

$$S = \int (kdx - \omega dt)$$

$$\delta S = \int \delta k(dx - \tfrac{\partial \omega}{\partial k} dt) + k\delta x| - \int \delta x(dk + \tfrac{\partial \omega}{\partial x} dt) = 0$$

$$\Downarrow$$

$$\frac{dx}{dt} = \frac{\partial \omega}{\partial k} \qquad \frac{dk}{dt} = -\frac{\partial \omega}{\partial x}$$ Hamilton equations for rays

Convective stabilization Near the Core

The longitudinal wavenumber of a Langmuir wave evolves as

$$\frac{dk_{\|}}{dt} = -\frac{\omega_e}{2}\frac{\partial \ln n_e}{\partial z} \equiv -\frac{\omega_e}{2L_n}.$$

While crossing the resonant interval $\Delta k_{\|}$ (located near $k_{\|} \approx \omega_e / c$), the wave amplitude undergoes

$$\Lambda \approx 2L_n \int_{\Gamma(\vec{k})>\nu} [\Gamma(\vec{k}) - \nu]\, dk_{\|} / \omega_e$$

exponentiations. If Λ_0 exponentiations can be tolerated, the stabilization condition is

$$L_n \le \Lambda_0 \omega_e / 2 \int_{\Gamma(\vec{k})>\nu} [\Gamma(\vec{k}) - \nu]\, dk_{\|} \equiv L_{nc}.$$

Numerical example

Transport stability condition near the core

$$\text{For } \Lambda_0 = 7$$

$$\max_{\vec{k}} \Gamma(\vec{k}) \equiv \Gamma \approx 2\nu \approx 10^{15} \text{ sec}^{-1}$$

the condition for the convective stabilization of the beam transport at nearly core concentration is

$$L_n \le L_{nc} \sim 50\ \mu\text{m}$$

Stopping Precisely in the Pellet Core

For realistic fast igniter parameters, ultra-relativistic electron beams can excite in the core Langmuir turbulence; this turbulence is potentially capable of significant decelerating these beams within the core.

Moreover, transport of the beams near the core might be convectively stabilized by natural density gradients.

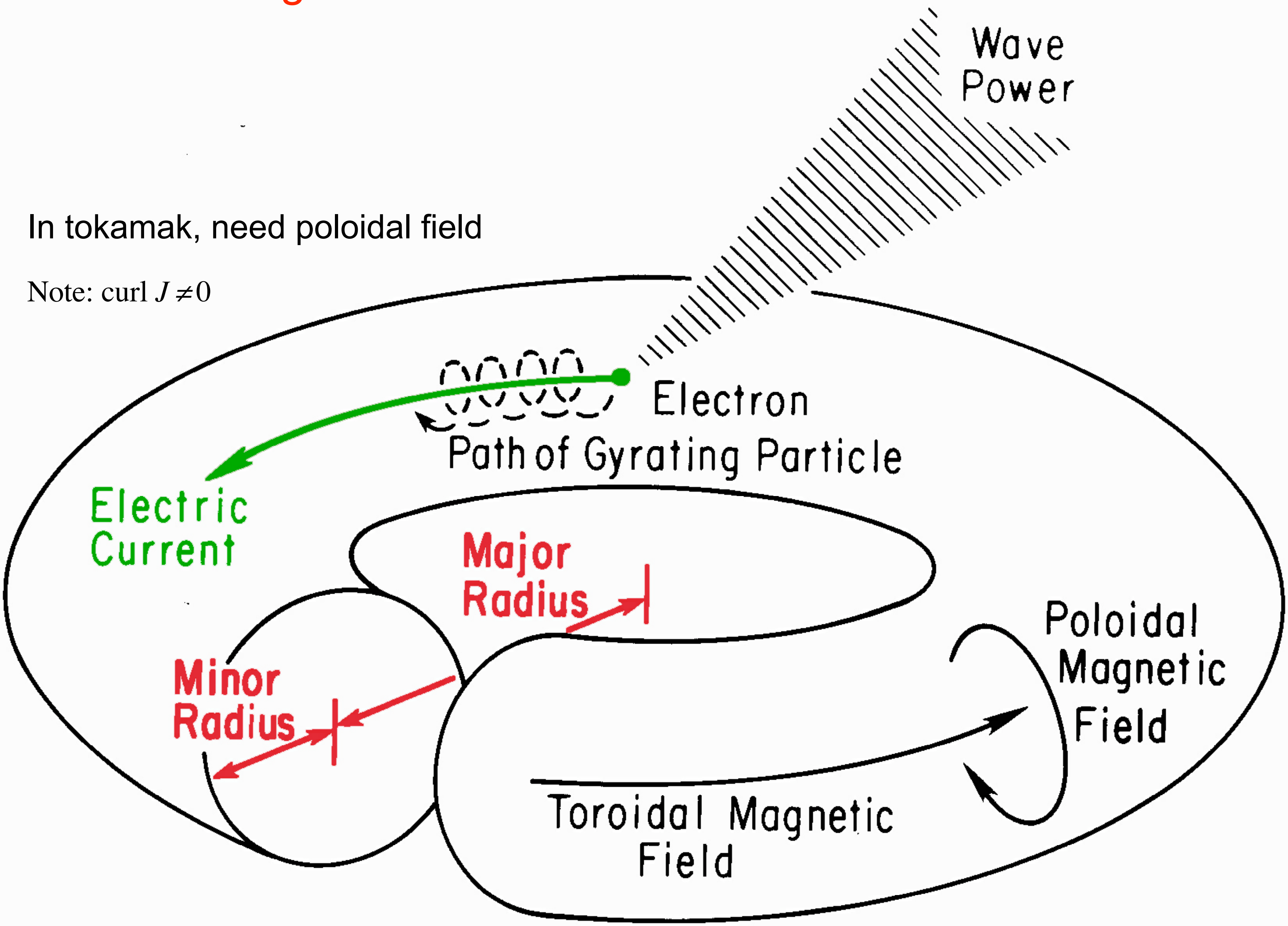
Malkin and Fisch, PRL, 2002

Q. How will collective effects in warm dense matter regimes be modified in strong density gradients?

Driving Current with Waves in Ideal Plasma

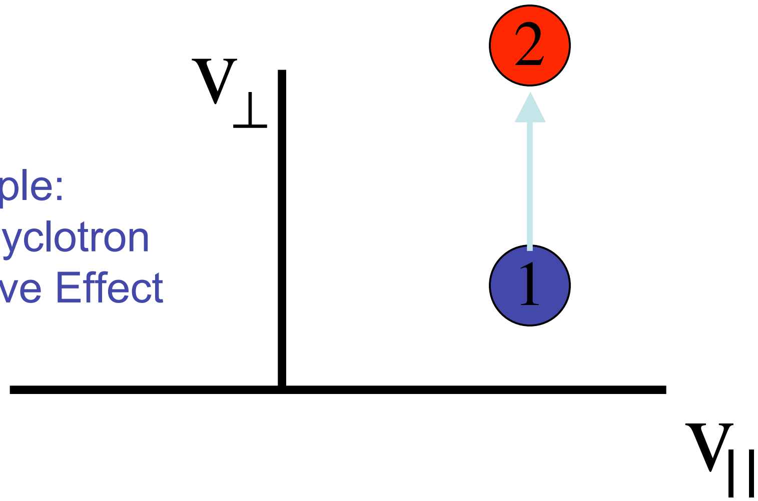
In tokamak, need poloidal field

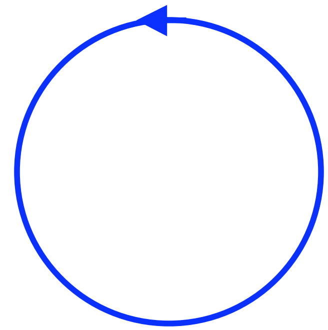
Note: curl $J \neq 0$



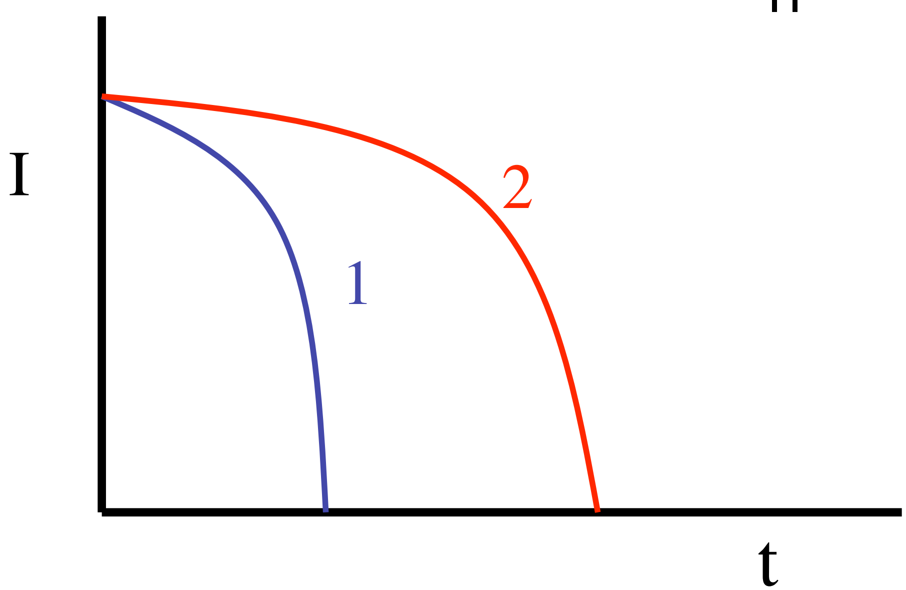
Key element: use optimum part of phase space

Example:
Electron Cyclotron
Current Drive Effect





Cyclotron motion

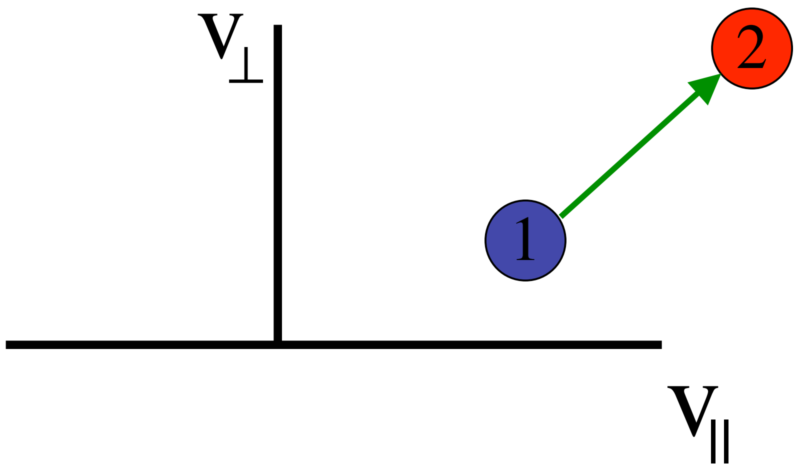


$$\omega - \vec{k} \cdot \vec{v} = n\Omega$$

$$\Omega \equiv eB/m$$

Fisch and Boozer, 1980

Generalized RF Current Drive Effect



$$\omega - \vec{k}\cdot\vec{\mathrm{v}} = n\Omega$$

$$J = \int d^3\mathrm{v}\ \overline{S}\cdot\overline{\nabla}\,\Psi$$

$$P_D = \int d^3\mathrm{v}\ \overline{S}\cdot\overline{\nabla}\varepsilon$$

$$\varepsilon \equiv m\mathrm{v}^2/2$$

New "transport quantity"
for current drive

$$\Psi = \int_0^\infty e v(t)\,dt$$

$$\boxed{\Psi(\mathrm{v}_\parallel, \mathrm{v}_\perp) = \frac{-e\,\mathrm{v}_\parallel \mathrm{v}^3}{\nu_0(5+Z_i)}}$$

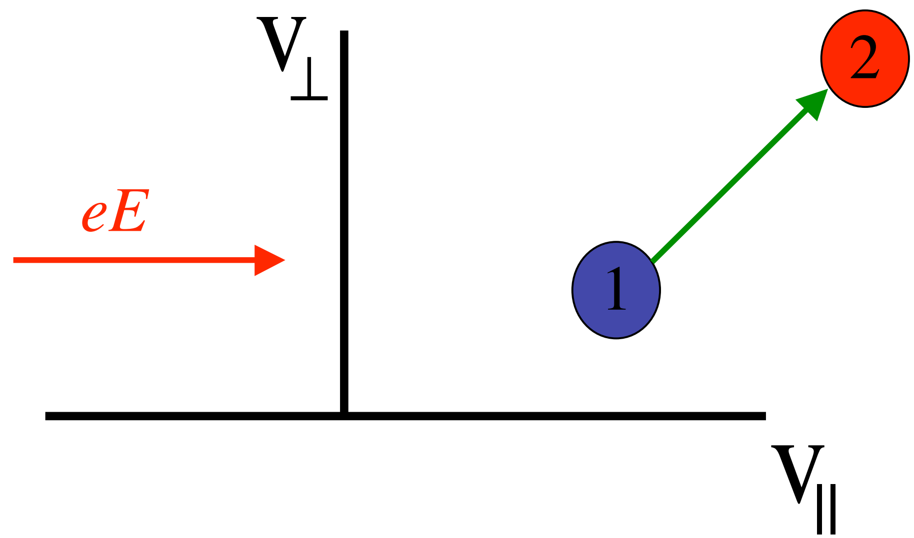
Green's function: Fisch and Boozer, 1980
Adjoint formalism: Antonsen and Chu, 1982

Generalized Transport Quantities

Associate transport quantity with each point in 2D velocity space

1. Current Drive Efficiency

   Generalizes Spitzer conductivity



2. Runaway Probability

   Generalizes Dreicer velocity

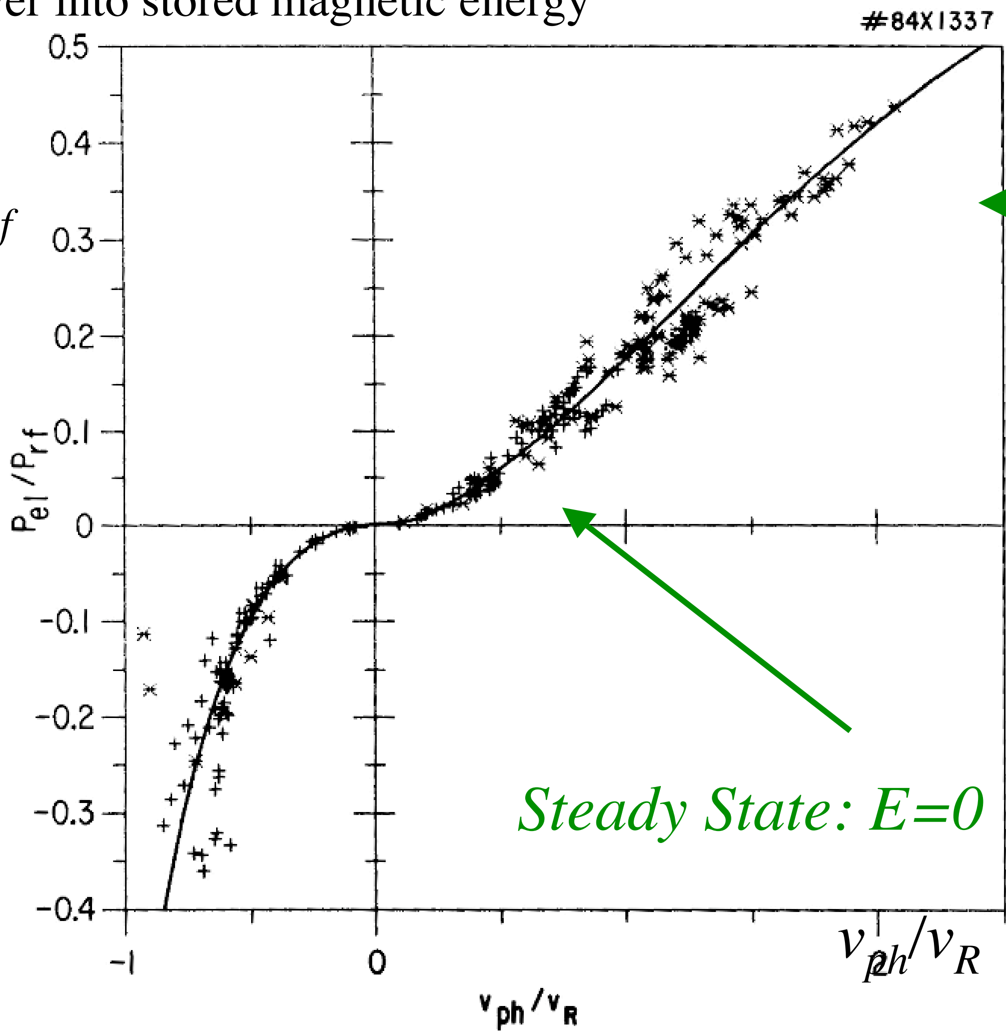
3. Energy flow to stored magnetic energy $$W_{el} = \int_0^\infty ev \cdot E\,dt$$

Fisch (1985); Fisch and Karney (1985); Karney and Fisch, 1986

Theory and Demonstration of the Ideal Plasma Current Drive Effect (with PLT Data)

P_{el} is power into stored magnetic energy

P_{el}/P_{rf}



v_{ph}/v_R

Ramp-up: E<0

Steady State: E=0

Karney, Fisch and Jobes (1985)

Corroborated on:

ASDEX: Leuterer (1991)
PBX: Giruzzi; Bernabei (1997)
HT-7: Chen (2005)

Also note ECCD consistency
DIII-D: Petty (2002)

v_R = runaway velocity

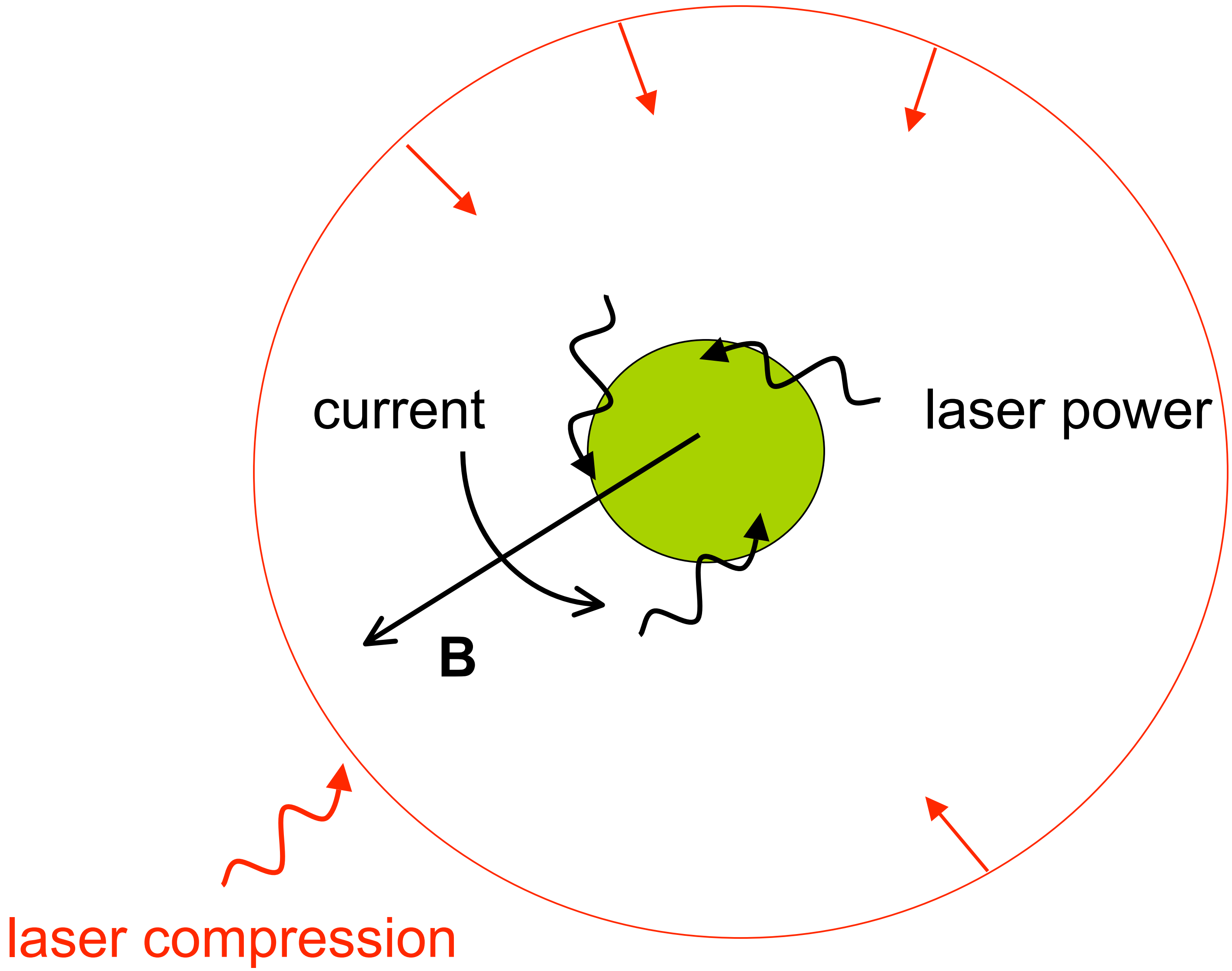
Verification *in detail* proves classical picture of electron collisions!

“Ideal Plasma” Current Drive Lessons

Larger Problem for Warm Dense Matter Regime
-- develop multi GG fields at high efficiency
-- need WDM, many electrons, low pressure,
(advantage in Fermi degenerate)

Fundamental physics: advance understanding not by considering integrated transport quantity like conductivity, but by by considering microscopic transport quantities.

Magnetic Field Generation



Energy in
GG in mm^3
Is 4 MJ

Elements of High-B Generation

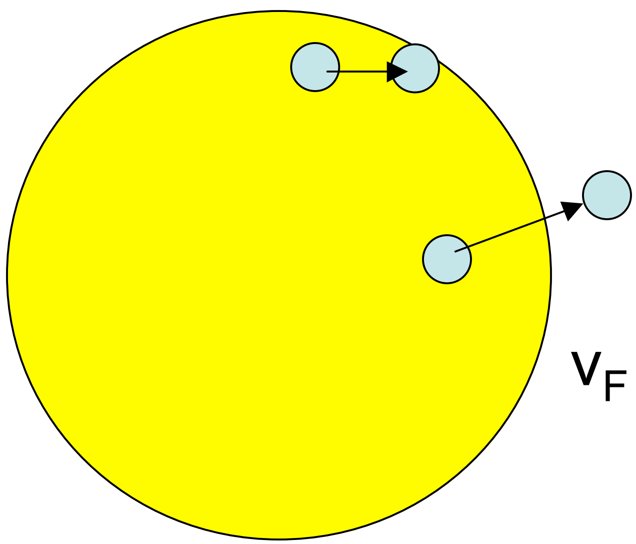
1. need high electron density
2. need high power density

Consider high density plasma (low temperature). Generate current over greater than L/R time. Compress in less than L/R time.

Efficiency of steady state current in Fermi degenerate plasma can be exploited.

$$I_{10} = \Theta(v_z / v_F) E_{100} R_{40} n_{24}^{1/3}$$

$$B = 2(I_{10} / r_{10}) GG$$

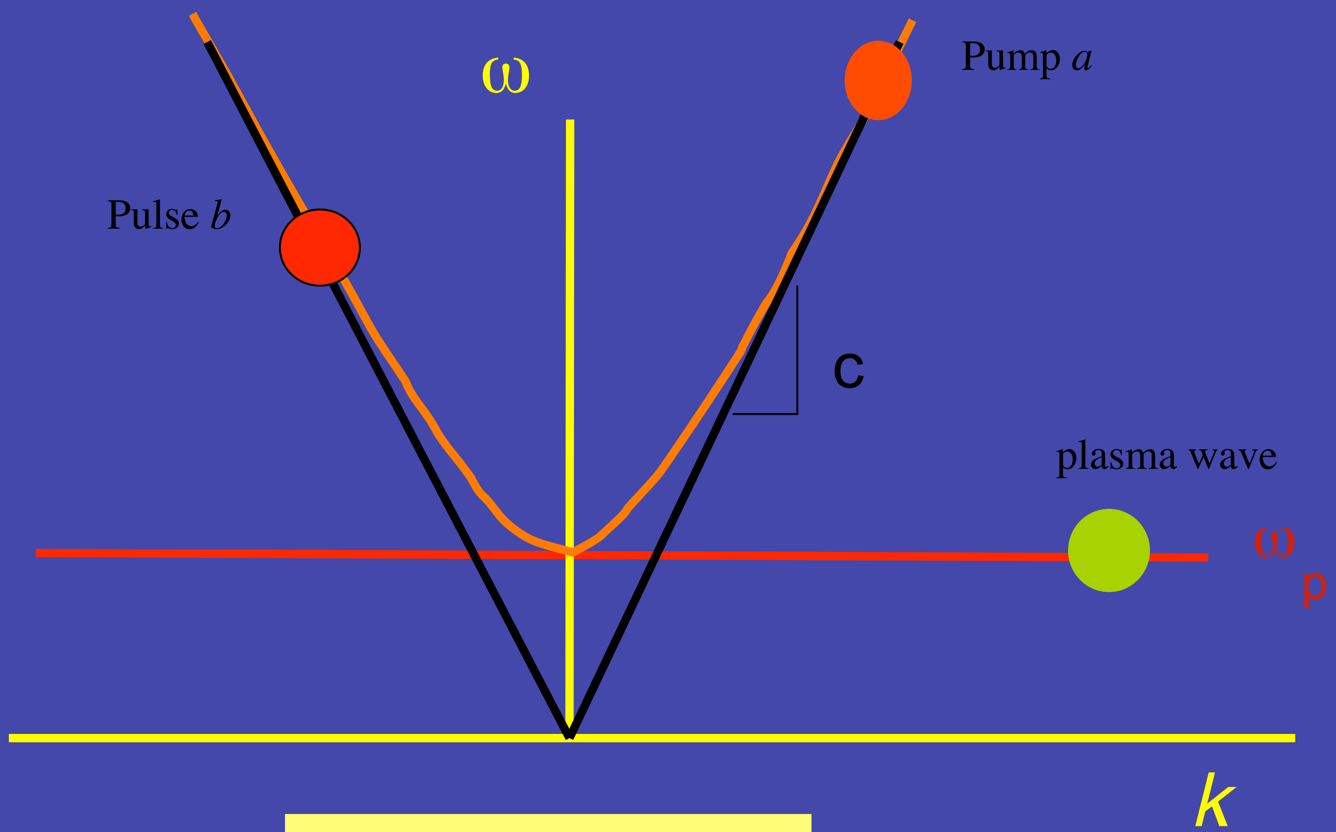


in units of 10 MA, 100 kJ, 40 μ

If L/R =0.5 ns, n_{24}= 10, r_{10}= 1. R_{40}= E_{100}= 1
Then for v_z=v_F,
I_{10}= 0.15, and B = 300 MG.

Son and Fisch (PRL, 2005)

Raman Decay in "Ideal" Plasma

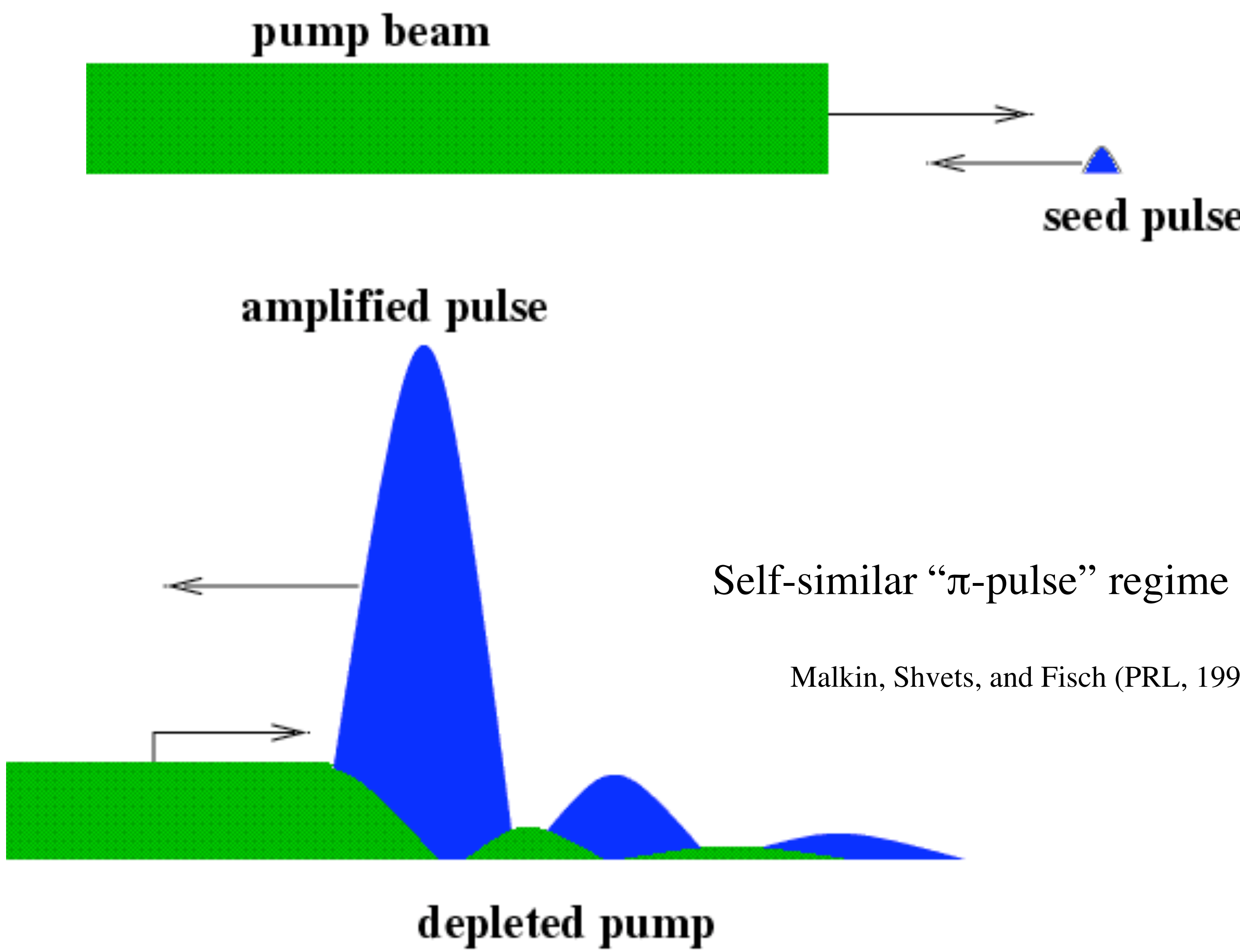


resonance condition

$$\omega_a - \omega_b = \omega_p$$

$$\vec{k}_a - \vec{k}_b = \vec{k}_p$$

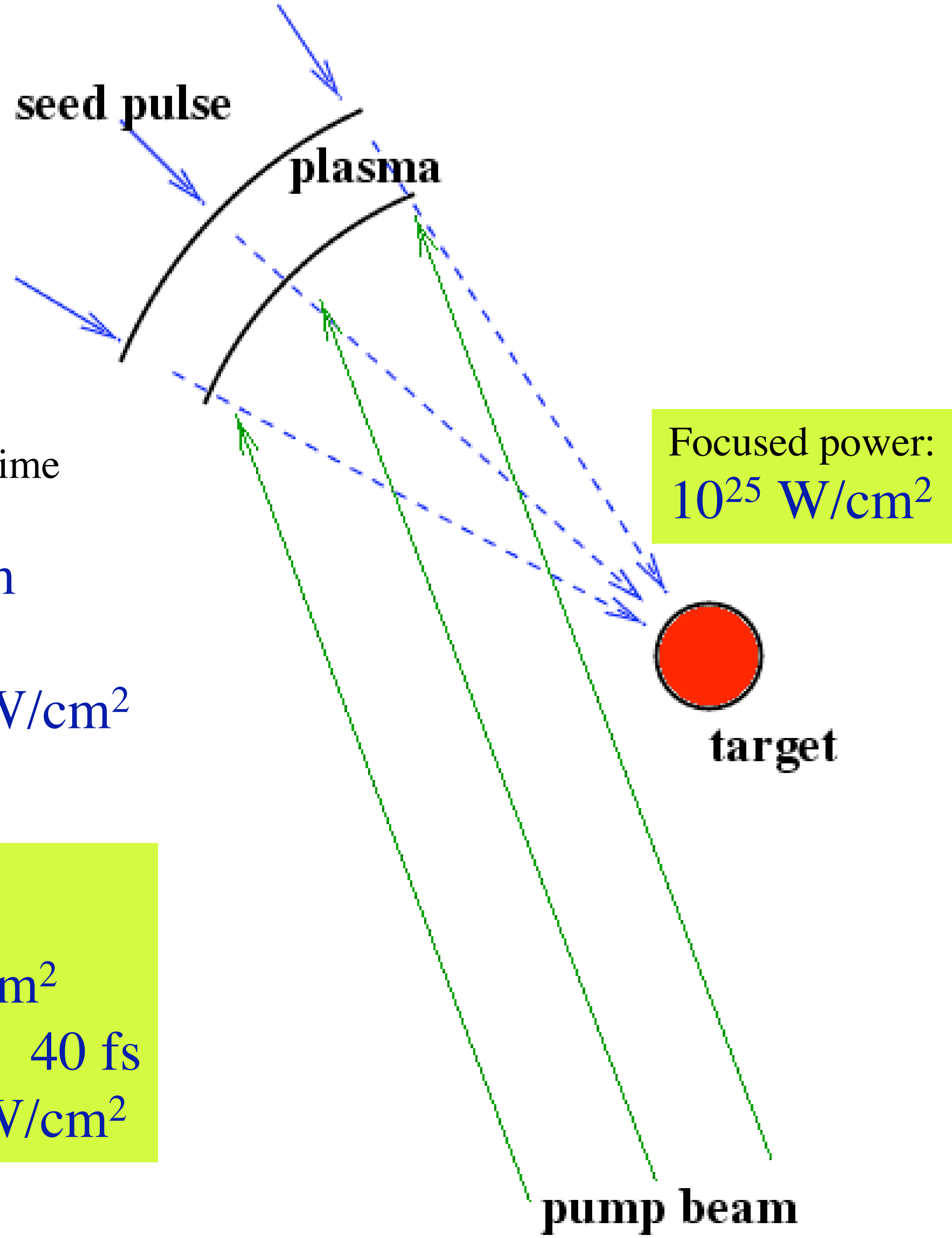
Amplification by Resonant Raman Backscatter in “Ideal Plasma”



Self-similar “π-pulse” regime

Malkin, Shvets, and Fisch (PRL, 1999)

Plasma as an Amplifier



Representative parameters
For Resonant Raman Backscatter Regime

Plasma width	0.7 cm
Pump duration	50 ps
Pump intensity	10^{14} W/cm^2

Output parameters (unfocused)

Pulse fluence	4 kJ/cm^2
Pulse duration	40 fs
Pulse power	10^{17} W/cm^2

Laser energy vacuum breakdown

Suppose critical electric field E_c =1.3 × 10^{16} V/cm^2

Suppose laser compression and focusing to laser wavelength λ

then the energy located within the volume λ^3 would be

$$\mathcal{E}_c \sim \lambda^3 E_c^2/8\pi \sim \lambda^3\, 8 \times 10^{18}\ \mathrm{J/cm^3}$$

λ	1 μm	100 nm	10 nm	1nm	1 Å
$\mathcal{E}_c$	8 MJ	8 kJ	8 J	8 mJ	8 μJ

Use MJ optical or mJ x-ray lasers?

Malkin, Fisch and Wurtele (PRE, 2007)

MJ optical or mJ x-ray lasers?

- The energy in either of these emerging laser systems would be, in principle, sufficient for producing vacuum breakdown intensities.

- LCLS (Linac Coherent Light Source) might have some energetic advantages over NIF (National Ignition Facility) or LMJ (Laser Megajoule).

- However, it would be necessary to compress the output laser pulses of either system to several wavelengths and to focus the compressed pulses to spot sizes of no more than several wavelengths.

Laser energy $\mathcal{E}$ vs $\mathcal{E}_c$ for the biggest of currently built lasers

Device	NIF or LMJ	LCLS
λ	0.35 µm	0.15 nm
$\mathcal{E}$	2 MJ	2 mJ
$\mathcal{E}_c$	1/3 MJ	1/40 mJ
$\mathcal{E}/\mathcal{E}_c$	6	80

Pulse Compression in “Transient” Medium: Relic Crystal Lattice

Consider plasma produced by the sudden ionization of crystals may be less uniform, because thermal ion motion would not smooth the ion lattice within the ultrashort x-ray pulse durations.

For instance, for a 0.3 fs pulse, the ion temperature needed for the lattice smoothing would be about A keV, where A is the atomic weight of ions (A=1 for hydrogen).

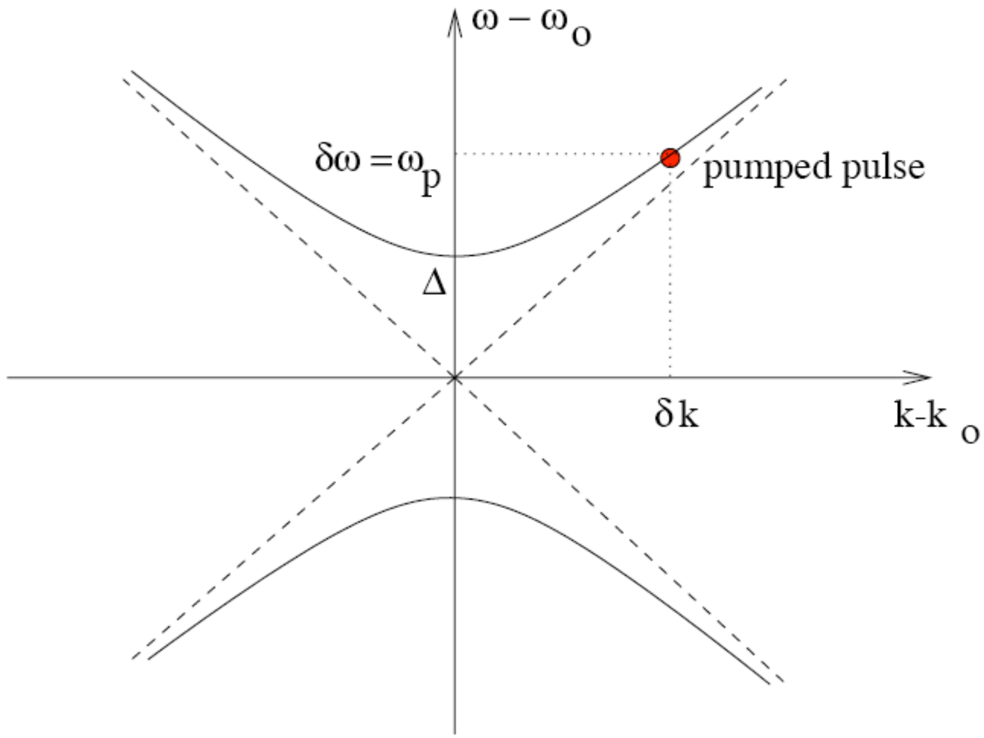
Such ion temperatures are not readily achievable, since it is the electron plasma that is primarily heated by the x-ray pulses, and the energy exchange between electrons and ions is relatively slow.

Neither does the longer-scale electric field produced from the ponderomotive evacuation of the electrons smooth (rather than just deform) the lattice, and a pulse of 0.3 fs duration is 100 nm long.

Useful effects of the relic crystal lattice

The x-ray frequency band gaps can suppress parasitic Raman scattering of amplified pulses;

Enhanced dispersion of the x-ray group velocity near the gaps can delay self-phase modulation instability, thereby enabling further amplification of the x-rays.



typical near-gap dispersion law

Malkin and Fisch, PRL, 07

Summary

Extreme Parameters Enabled by Warm Dense Plasma
-- and Mediated by Plasma Waves

1. Acceleration of charged particles through plasma wakes
    a. What are maximum gradients and energies?
    b. How long need a plasma wave persist?
2. Differential collisionless deceleration of charged particles
    a. Fast ignition with differential core stopping
3. Current Drive in Dense Plasma
    a. What is highest magnetic field that can be generated in the laboratory?
    b. Pose Adjoint Problem -- then integrate to get conductivity!
4. X-ray pulse compression and focusing; fs to as
    a. What are the largest x-ray intensities that can be obtained and focused -- vacuum breakdown?
    b. How long need a plasma wave -- or medium -- persist?

[redacted]